Math 472: Homework 08

Due Tuesday, March 31

Problem 1 (Exercise 9.82)**.** Let $Y_1, \ldots, Y_n$ be a random sample from the density function given by

$$f(y \mid \theta) = \begin{cases} \left(\frac{1}{\theta}\right) r y^{r-1} e^{-y^r/\theta} & : \quad \theta > 0, y > 0 \\ 0 & : \quad \text{elsewhere.} \end{cases}$$

where $r > 0$ is a known positive constant.

(a) Find a sufficient statistic for θ.

(b) Find the MLE of θ.

Problem 2. Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x \mid \theta) = \theta x^{-2}, \quad 0 < \theta \leq x < \infty.$$

(a) Find a suffificent statistic for θ.

(b) Find the MLE of θ.

Problem 3 (Shifted exponential)**.** A water supply becomes contaiminated at some unknown time μ.

Suppose there are a total of n sensors in the water. After the contamination, these sensors don't immediately start testing positive, but more realistically do so with with some lag times (owing to sensor location, sensitivity, and uneven contaminant concentrations).

For each $i \in [n]$, let X_i be the first time that sensor i tests positive for contamination. Assume that $X_1, \ldots, X_n$ are independent and each have distribution given by the pdf

$$f(x \mid \theta) = e^{-(x-\mu)} \mathbf{1}_{[\mu \leq x < \infty]}.$$

(a) Show that $X_{(1)} := \min(X_1, \ldots, X_n)$ is a sufficient statistic for μ.

(b) Find the MLE of μ.

Problem 4 (Exercise 10.2)**.** An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel her claims are inflated. In an attempt to disprove her claim, we administer the drug to 20 insomniacs and observe Y, the number for whom the drug dose induces sleep. We wish to test the hypothesis $H_0 : p = .8$ vs the alternative $H_a : p < .8$. Assume that the rejection region $\{y \leq 12\}$ is used.

(a) In terms of this problem, what is the type I error?

(b) Find α.

(c) In terms of this problem, what is the type II error?

(d) Find β when $p = .6$.

(e) Find β when $p = .4$.

Problem 5 (Exercise 10.3)**.** Refer to Problem 4.

(a) Find the rejection region of the for $\{y \leq c\}$ so that $\alpha \approx .01$.

(b) For the the rejection region in part (a), find β when $p = .6$.

(c) For the rejection region in part (a), find β when $p = .4$.

Problem 6 (Exercise 10.19)**.** The output voltage for an electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean 128.6 and standard deviation 2.1. Using the large-sample z-test method described in section 10.3 of the textbook, test the hypothesis that the average output voltage is 130 against the alternative that it is less than 130. Use a test with level .05.

Problem 7 (Exercise 10.37)**.** Refer to Problem 6. If the voltage falls as low as 128, serious consequences may result. For testing $H_0 : \mu = 130$ versus $H_a : \mu = 128$, find the probability of a type II error, β for the rejection region used in Problem 6

Problem 8 (Exercise 10.20)**.** The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of an alloy of steel, the Rockwell hardness index averaged 62 with standard deviation 8. The manufacturer claims that this alloy has an average hardness index of at least 64. Is there sufficient evidence to refute the manufacturer's claim at the 1% significance level? Use the large-sample z-test method described in section 10.3 of the textbook.

Problem 9 (Exercise 10.38)**.** Refer to problem Problem 8. The steel is sufficiently hard to meet usage requirements if the mean Rockwell hardness measure does not drop below 60. Using the rejection region found in Problem 8, find β for the specific alternative $\mu_a = 60$.

Problem 10 (Salk Polio Field Trial (1954))**.** The results of a 1954 polio vaccine trial in $n = 401,974$ elementary school students gave the following results:

	No Polio	**Polio**
Vaccinated	200,712	33
Unvaccinated	201,114	115

(1)

We wish to know if there is a relationship between vaccination status and the incidience of paralytic polio. The null hypothesis is:

$$H_0 = \text{vaccination and polio are independent}$$

and the alternative hypothesis is

$$H_a = \text{ there is a relationship between vaccination and polio.}$$

Conduct a hypothesis test using the chi-squared statistic to test these hypotheses.