Worksheet 2: Set Theory

Part 1: Set notation

Problem 1 (Reading a set)**.** Let

$$S = \{4, 1, 4, 2, 1, 3\}, \qquad T = \{3, 2, 4, 1\}.$$

(a) Rewrite S without repetitions and find $|S|$.

$S =$ $\quad$ $|S| =$

(b) Decide whether each statement is true or false:

$2 \in S$	**T**	**F**
$5 \in S$	**T**	**F**
$\{1, 2\} \subseteq S$	**T**	**F**
$\{1, 2\} \in S$	**T**	**F**

(c) Are S and T equal? Explain briefly.

Problem 2 (Set-builder notation)**.**

(a) List the elements of $\{n \in \mathbb{Z} : -3 \leq n \leq 3\}$.

(b) Describe the set $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ using set-builder notation.

(c) List the elements of $P = \{(x, y) : x \in \{1, 2\} \text{ and } y \in \{a, b, c\}\}$.

(d) Find $|P|$.

Problem 3 (Set operations)**.** Let

$$U = \{1, 2, 3, \ldots, 12\}, \qquad A = \{n \in U : n \text{ is even}\}, \qquad B = \{n \in U : n \text{ is divisible by } 3\}.$$

(a) List the elements of A and B.

$A =$ [ ]

$B =$ [ ]

(b) Find each of the following sets:

$A \cap B =$ [ ]

$A \cup B =$ [ ]

$A \setminus B =$ [ ]

$A^c =$ [ ]

$B^c =$ [ ]

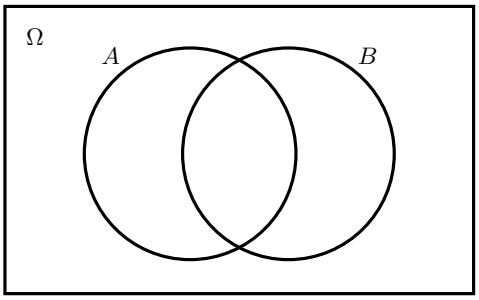
(c) Is $A \subseteq B$? Explain.

(d) Are A and B disjoint? Explain.

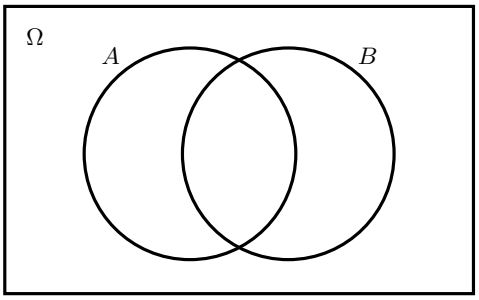
Part 2: Two important set-theoretic results

Problem 4 (De Morgan's laws)**.** Let A, B, Ω be sets. (The symbol Ω is called Omega, and is commonly used in probability). Suppose $A, B \subseteq \Omega$ (like shown in the below two pictures.)

(a) Shade $(A \cup B)^c$ in the first Venn diagram and $A^c \cap B^c$ in the second.



$(A \cup B)^c$



$A^c \cap B^c$

(b) Are the shaded regions the same? Complete the identity

$$(A \cup B)^c = \boxed{}$$

Problem 5 (The inclusion–exclusion formula)**.** The ***inclusion–exclusion formula*** states that if A and B are finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

Explain why this identity holds using pictures.

Part 3: Connection to probability theory

Problem 6 (Dice sets)**.** Suppose we roll two dice. We represent the outcome by an ordered pair (x, y), where x is the result of the first dice and y is the result of the second. The set of all possible outcomes is

$$\Omega = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}.$$

(a) How many elements does λ contain? Explain how you can count them without listing every element.

(b) Let B be the set of outcomes for which the sum of the two dice is 7. Describe B in set-builder notation, and then list its elements.

(c) Let C be the set of outcomes for which the two dice roll the same number. Describe C in set-builder notation, and then list its elements.

(d) Describe the following three sets in words. What events do they represent?

$$D = \{(x, y) \in \Omega : x > y\}, \qquad E = \{(x, y) \in \Omega : x = 6 \text{ or } y = 6\}, \qquad F = \{(1, 1)\}.$$

(e) In probability we use the word "event" rather than "set". Describe the following events in set-builder notation:

(i) The event that the dice add up to at least 10.

(ii) The event that at least one dice shows an even number.

(f) List the elements of the following sets

$B \cap E =$ [ ]

$C \backslash E =$ [ ]

(g) Compute the following probabilities:

$\mathbb{P}(D) =$ [ ] $\mathbb{P}(E) =$ [ ]

$\mathbb{P}(F) =$ [ ] $\mathbb{P}(B \cap E) =$ [ ]