Math 243: Homework 09

Due Friday, November 6

Problem 1.

(a) Find the line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t$, $y = 6 - 3t$, $z = 3 + 9t$.

(b) Find the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

Problem 2.

(a) What does the equation $y = x^2$ represent as a curve in $\mathbb{R}^2$?

(b) What does it represent as a surface in $\mathbb{R}^3$?

(c) What does the equation $z = y^2$ represent?

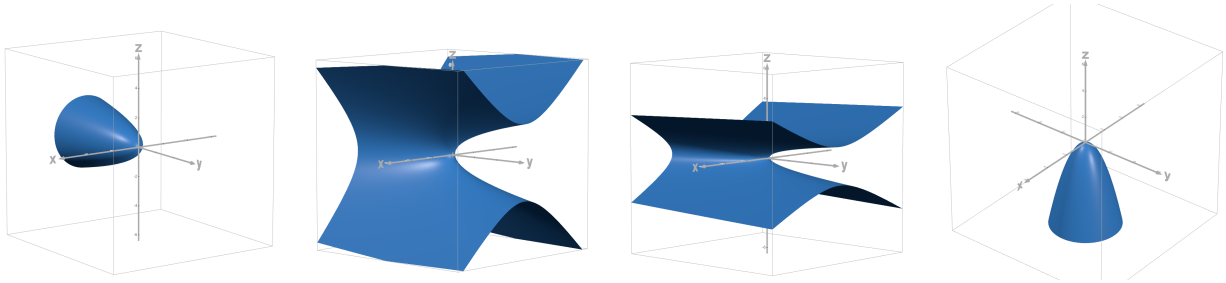
Problem 3. By consideration of the traces (i.e., cross sections), match the equations with their pictures below. Justify your answers.

(a) $x^2 - 4z^2 + y = 0$

(b) $x^2 + z^2 + y = 0$

(c) $x^2 + z + y^2 = 0$

(d) $x^2 - z^2 + y = 0$



Problem 4. Find the domain of the following functions:

(a)

$$\vec{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$$

(b)

$$g(x, y) = \frac{\ln(2-x)}{1-x^2-y^2}$$

Problem 5. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter t

(a) $\vec{r}(t) = \langle \arctan(t), 2e^{2t}, 8te^t \rangle$ at $t = 0$

(b) $\vec{r}(t) = \langle \sin^2(t), \cos^2(t), \tan^2(t) \rangle$ at $t = \pi/4$

Problem 6. Find the length of the curve

$$\vec{r}(t) = \langle 1, t^2, t^3 \rangle, \quad 0 \le t \le 9$$

Problem 7. Find the curvature of $\vec{r}(t) = \langle 0, t^3, t^2 \rangle$

Problem 8. A projectile is fired with an inital speed of 200m/s at an angle of 60°. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact. Round your final answers to the nearest meter.

Problem 9. Find the limit, if it exists, or show that the limit does not exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$$

(c)

$$\lim_{(x,y)\to(0,0)} \frac{5y^4 \cos^2(x)}{x^4 + y^4}$$

Problem 10. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find exact length of C from the origin to the point $(6, 18, 36)$.