Math 243: Homework 08

Due Wednesday, Oct 28

To recieve full credit, it is necessary that you show your work.

Problem 1. Find the parametric equation of the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$.

Problem 2.

(a) Find parametric equation for the line through $(2, 4, 6)$ that is perpendicular to the plane given by the equation $x - y + 3z = 7$.

(b) In what points does this line interect the coordinate planes?

Problem 3 (Orthogonality on a circle)**.** Let $A = (-1, 0)$ and $B = (1, 0)$, and let $C = (x, y)$ be any point on the unit circle. In this problem will show the remarkable fact the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are always orthogonal.

(a) Sketch the setup of the problem (e.g., the unit circle, the points A and B, an arbitrary point (x, y) on the unit circle, and the two vectors).

(b) Compute the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$.

(c) Show that $\overrightarrow{CA}$ and $\overrightarrow{CB}$ form a right angle.
(Hint: the fact that the point (x, y) is on the unit circle tells us that $x^2 + y^2 = 1$.)

Problem 4. Let $\mathbb{P}$ be the plane passing through the point $(1, -1, -1)$ which is parallel to the plane $5x - y - z = 6$. What is the normal form equation of $\mathbb{P}$?

Problem 5. What is the equation of the plane which passes through the points $A = (3, 0, -1)$, $B = (-2, -2, 3)$, and $C = (7, 1, -4)$?

Problem 6. Use intercepts to sketch the plane $6x + 5y - 3z = 15$.

Problem 7. Find the distance between the point $X = (1, -2, 4)$ and the plane given by the equation $3x + 2y + 6z = 5$.

Problem 8. Consider the line L_1 given by parametric quation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad \text{with } -\infty < t < \infty$$

and the line s

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad \text{with } -\infty < s < \infty$$

(a) At what point do the lines L_1 and L_2 intersect?

(b) Find an equation of the plane that contains these two lines.

Problem 9. Find a vector equation for the line segment between the points $(6, -1, 9)$ and $(7, 6, 0)$.